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14. ABSTRACT
The objective of the project is to develop techniques to allow precise tracking of GPS signals in the presence of multipath interference. Our work has focused specifically on a difficult class of multipath whose time delay relative to that of the direct line of sight signals (DLOS) is very short. This form of multipath is difficult to detect and mitigate using traditional correlator-based methods. Our approach overcomes the limitations of the existing methods by exploiting the spatial diversities between the DLOS and multipath sources. This approach requires the use of a receiver array and batch-based software receiver processing. The spatial diversity between the multipath and DLOS GPS signals makes it possible to estimate the angle of arrival (AOA) for each signal. Knowledge of the source AOAs makes it possible to reconstruct the pseudo-inverse for the array manifold and to recover DLOS and multipath signals. During the three year funding time period, we successfully developed and implemented algorithms for detection, estimation, and mitigation of the short delay time multipath signals.
In this report, we will give an overview of the technique, the detection, estimation, and mitigation algorithms, and performance

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Precise GPS Signal Tracking in Interference and Multipath Environment Using a Multi-Channel Software Receiver

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1. Technical Summary

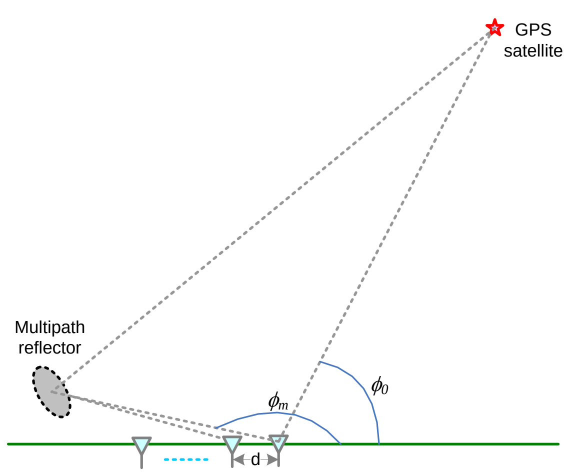
The objective of the project is to develop techniques to allow precise tracking of GPS signals in the presence of multipath interference. Our work has focused specifically on a difficult class of multipath whose time delay relative to that of the direct line of sight signals (DLOS) is very short. This form of multipath is difficult to detect and mitigate using traditional correlator-based methods. Our approach overcomes the limitations of the existing methods by exploiting the spatial diversities between the DLOS and multipath sources. This approach requires the use of a receiver array and batch-based software receiver processing. The spatial diversity between the multipath and DLOS GPS signals makes it possible to estimate the angle of arrival (AOA) for each signal. Knowledge of the source AOAs makes it possible to reconstruct the pseudo-inverse for the array manifold and to recover DLOS and multipath signals. During the three year funding time period, we successfully developed and implemented algorithms for detection, estimation, and mitigation of the short delay time multipath signals.

In this report, we will give an overview of the technique, the detection, estimation, and mitigation algorithms, and performance evaluations of the method.

1.1. Background Research and Technical Approach Overview

Multipath is a challenging error source in GPS range measurements [1]. Many algorithms and methods have been developed to mitigate multipath errors. Despite the differences in detailed implementations of these methods, they are all based on the exploitation of the temporal, spectral, spatial, and/or polarization diversity among the direct line-of-sight (DLOS) and multipath signals [2-8]. For multipath with short delay times, these existing methods are not effective [9]. For high precision GPS applications, the short delay time multipath cannot be ignored, even when the multipath signal is relatively weak. Simulations show that even if a multipath delay is in the order of a few ns and the multipath signal power is over 20dB below that of its DLOS, the multipath may still generate code phase errors in the order of tens of cm [10].

We proposed and implemented an approach that exploits the spatial diversities of the multipath a DLOS signals by processing a software GPS receiver array raw radio frequency inputs. The approach involves three distinctive stages of processing: multipath detection using an ANOVA-based hypothesis test, DLOS and multipath signal AOA estimation using an efficient contractive map and alternating projections, and recovering of the DLOS signal using pseudo-inversion. Figure 1 shows the schematics of an ideal linear receiver antenna array assumed for this study and the overall software receiver signal processing functions.



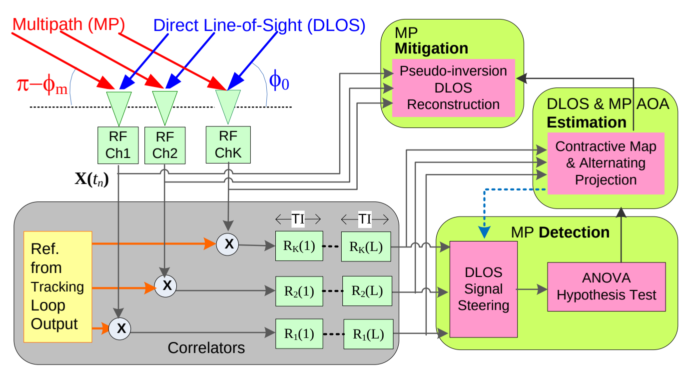


Fig. 1 Linear antenna array and high level schematics of the multipath processing algorithm

1.2. ANOVA-Based Multipath Detection

We developed a statistical detection test for short delay time GPS multipath based on a one-way ANOVA method. Description of this method can be found in [10]. A brief summary is provided here.

This method is derived based on three assumptions: the receiver is in tracking mode, the DLOS GPS signal AOAs are known and are different from their multipath AOAs. The second assumption will be removed after we discuss the AOA estimation for DLOS and multipath in the following section. The detection algorithm consists of two distinctive components as shown in figure 1: DLOS signal steering and ANOVA hypothesis test.

The objective of DLOS signal steering is to remove the relative phase difference of DLOS GPS signal from each receiver channel input. This is done by correlating the signal from each channel with a reference signal derived from any one of the receiver channel tracking results. The expected values of the correlations from the channels should be statistically identical if and only if no multipath is present. Based on this principal, multiple correlations over several successive CA code time blocks are performed to obtain and KxN correlation outputs where K the number of antennas and L is the number of correlation blocks for each channel.

The ANOVA hypothesis test is performed in the KxL correlation outputs. This is equivalent to the problem of having K disjoint populations, each having N random samples. The samples within each population follow a normal distribution. The ANOVA test determines if the means of all K populations are the same. The hypothesis test is based on an F-parameter which is defined as the following:

$$F_\alpha = \frac{MSE_B^{(\nu_1)}}{MSE_A^{(\nu_2)}} \tag{1}$$

where MSE_A is the average of sample variance within each population, while MSE_B is the variance between the means of the K populations. MSE_A follows a central chi-squared random distribution with K(N-1) degrees of freedom. The distribution of MSE_B depends on which hypothesis holds. The hypothesis test is:

$$\begin{aligned} &\text{Under } H_0 : \frac{MSE_B}{MSE_w} \sim F\left(2(K-1), 2K(L-1)\right) \\ &\text{Under } H_1 : \frac{MSE_B}{MSE_w} \sim F\left(2(K-1), 2K(L-1), \Lambda\right) \end{aligned} \tag{2}$$

Under H_0, the MSE_B has a central chi-squared distribution whose number of degrees of freedom is equal to K-1. Under H_1, MSE_B has a non-central chi-squared distribution with K-1 degrees of freedom. Under H_0, the ratio of MSE_B to MSE_A (ie, F_α) has a central F distribution whose numerator and denominator degrees of freedom are equal to v_1=K-1 and v_2=K(L-1), respectively. Under H_1, F_α is distributed as a non-central F-distribution (having the same degrees of freedom as its central counterpart). Since the probability density of the non-central distribution is right shifted relative to that of the central F-distribution, a decision rule to decide which distribution the ratio comes from (and therefore which hypothesis is true) can be constructed.

One disadvantage of working with the F distribution is the need for a look up table to compute F_α. This is a more troublesome problem because the denominator degrees of freedom (v_2=K(L-1)) varies if the number of samples L within each population changes (which happens in adaptive signal processing applications). We can avoid using a look up table for the critical values of the central F-distribution by using an approximation. This approximation, which relates F_α to the value of the standard normal distribution Z_α at which the CDF of distribution equals to α, is given by:

$$Z_\alpha \approx \frac{(1-B)\left(F_\alpha\right)^{1/3} - (1-A)}{\sqrt{B\left(F_\alpha\right)^{2/3} + A}} \tag{3}$$

This approximation was presented in [11]. The symbol A and B are related to the MSE_A and MSE_B degrees of freedom. From this approximation, we can solve for F_α:

$$F_\alpha = \left\{ \frac{ab + \sqrt{(A - a^2)(b^2 - B) + (ab)^2}}{(b^2 - B)} \right\}^3 \qquad (4)$$

with a = (1-A)/Z_α and b = (1-B)/Z_α.

We apply this problem to GPS multipath detection. Ideally, if there is no multipath, then the sample mean among the channels should be identical, hence the null hypothesis. In practice, we can set a critical value which is dependent on some user specified detection criteria and the hypothesis test is performed against this critical value.

Simulations demonstrated that the use of spatial diversity dramatically improves the performance of this method as the temporal delay between the GPS and multipath signals decreases. This characteristic complements that of existing multipath detection and mitigation methods. Figure 2 compares the missed detection date and false alarm rate (FAR) relationship generated using Monte-Carlo simulations and predicted by the ANOVA test. The multipath signal used in the test is 10 dB below its DLOS signal. The AOAs of the multipath and the DLOS are only 5 degrees apart from each while delay time between the two is set to 0, the worst possible scenario. The linear array has 5 elements and five code blocks of data were used to generate and 5x5 data matrix. The ANOVA test and the simulation results completely agree with each other. Additional performance tests on the multipath and DLOS AOA distributions, multipath signal power, multipath and DLOS relative delay time, antenna array dimensions, and number of data blocks used in correlations were also performed. Detailed results can be found in [10].

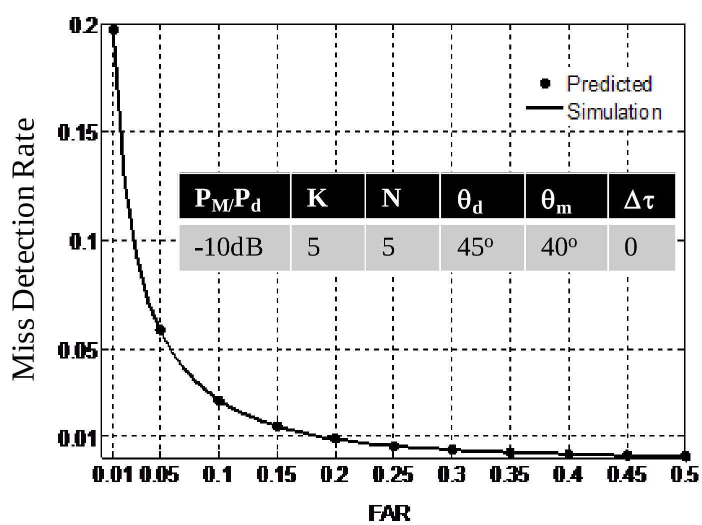


Fig. 2 Comparison between the ANOVA hypothesis predicted missed detection and false alarm rate with Monte Carlo simulation results.

1.3. AOA Estimation Using A Contractive Map and Alternating Projections

The contractive map is an iterative MLE designed to efficiently and accurately compute a signal's AOA. The concept and basic mathematical formulations for the contractive map has been presented in [10,12]. A brief summary is provided here.

Consider the case where there is only one signal (such as the DLOS GPS signal) in the input, the N consecutive batches of correlator outputs from the K channels can be averaged to generate the following vector:

$$\mathbf{Y} = \rho \mathbf{S}(\gamma) + \mathbf{e} \tag{5}$$

ρ and $\mathbf{e}$ are the average signal cross-correlation coefficient and post-correlated channel noise:

$$\mathbf{e} \sim \mathcal{N}_c\left(0, \frac{\sigma^2}{N \times K}\mathbf{I}_K\right) \tag{6}$$

Detailed analysis in [12] show that the MLE for γ is:

$$\gamma_{MLE} = \underset{\gamma \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}{ArgMax}\{\left|\mathbf{Y}^H \mathbf{S}(\gamma)\right|^2\} \tag{7}$$

The MLE can be solved iteratively with an arbitrary initial value $\gamma=\gamma$□$[0]$. [12] explains the detailed derivation for the MLE solutions:

$$\gamma[j] = \gamma[j-1] + \sqrt{K}\,\frac{A - CD + \sqrt{(A-CD)^2 + DB^2}}{DB} \tag{8}$$

where,

$$\begin{aligned} A &= \left|\mathbf{Y}^H \mathbf{S}'\right|^2 \qquad B = 2\,\mathrm{Re}\{(\mathbf{Y}^H \mathbf{S}')(\mathbf{S}^H \mathbf{Y})\} \\ C &= \left|\mathbf{Y}^H \mathbf{S}\right|^2 \qquad D = \frac{K^2 - 1}{3} \end{aligned} \tag{9}$$

The MLE solution is then converted to the signal AOA solution:

$$\begin{aligned} \phi &= \cos^{-1}\left(\frac{2}{\pi}\gamma_{final}\right) \\ \gamma_{final} &= \begin{cases} \gamma[j] & if\ |\gamma[j]| \le -\pi/2 \\ \mathrm{mod}_{\pi}(\pi + \gamma[j]) & if\ |\gamma[j]| > \pi/2 \end{cases} \end{aligned} \tag{10}$$

Figure 3 plots the MLE and illustrates the performance of the contractive map for a 7-element linear array antennas. A signal AOA is at 140 degree with a nominal GPS satellite SNR at -18 dB. The initial AOA estimation is at 80 degree. It took 4 iterations for the AOA solution to converge to a solution satisfying our preset estimation accuracy.

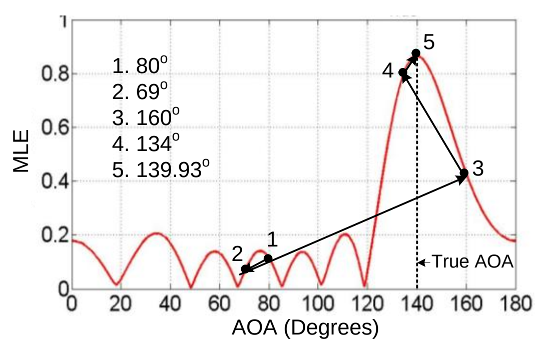


Figure 3. Illustration of a contractive map simulation example.

The contractive map only solves for one signal source AOA. If multiple signal sources are present such as in the case of the GPS multipath problem, the contractive map is incorporated with the alternating projection method to provide AOA estimations for multiple sources. The alternating projection method holds all except one of the AOAs as fixed quantities. The one AOA is allowed to vary in such a way as to optimize the likelihood function. A single cycle consists of all the AOAs being varied to achieve their corresponding optimization. Then the entire cycle is repeated until some pre-set criterion is satisfied. [10] presented detailed description of how to apply the alternating projection method to estimate multiple signal AOAs.

Assuming the number of multipath sources, M, is known, along with the noise variance, the MLE of $\boldsymbol{\gamma}=[\gamma_0, \gamma_1, \ldots, \gamma_M]$ must satisfy:

$$\boldsymbol{\gamma}_{MLE} = \underset{\boldsymbol{\gamma}\in\{\gamma_0\times\gamma_1\times\cdots\gamma_M\}}{ArgMax}\left\{\mathbf{Y}^H\left(\mathbb{S}(\gamma)\left(\mathbb{S}^H(\gamma)\mathbb{S}(\gamma)\right)^{-1}\mathbb{S}^H(\gamma)\right)\mathbf{Y}\right\} \tag{11}$$

$\mathbb{S}(\boldsymbol{\gamma})$ is a matrix whose columns are the steering vectors of each signal source:

$$\mathbb{S}(\boldsymbol{\gamma}) = [\mathbf{S}_0\,\mathbf{S}_1\cdots\mathbf{S}_M] \tag{12}$$

Without loss of generality, we hold all other signal AOAs as constants to find the MLE for one signal whose steering vector is $\mathbf{S}_0$. We partition the column space of $\mathbb{S}$ into two parts:

$$\mathbb{S}(\boldsymbol{\gamma}) = [\mathbb{S}_{+0}\,\mathbb{S}_{-0}] \tag{13}$$

where, $\mathbb{S}_{+0} = \mathbf{S}_0$, $\mathbb{S}_{-0} = [\mathbf{S}_1\cdots S_M]$.

Denote $\mathbb{S}_{-0}^{\perp 0}$ as the subspace of $\mathbb{S}_{-0}$ perpendicular to $\mathbb{S}_{+0}$, then:

$$Col\left(\mathbb{S}_{-0}^{\perp 0}\right) = \left(\mathbb{I}_K - \mathbb{P}_{Col(\mathbb{S}_{+0})}\right)\mathbb{S}_{-0} \tag{14}$$

The projection of the input onto to $\mathbb{S}(\boldsymbol{\gamma})$, $\mathbb{P}_{Col(\mathbb{S}(\boldsymbol{\gamma}))}$, can be decomposed into two parts:

$$\mathbb{P}_{Col(\mathbb{S}(\boldsymbol{\gamma}))} = \mathbb{P}_{Col(\mathbb{S}_{+0})} + \mathbb{P}_{Col(\mathbb{S}_{-0}^{\perp 0})} \tag{15}$$

The MLE with respect to γ_0 is:

$$\hat{\gamma}_0\Big|_{\gamma_1\times\gamma_2\times\cdots\gamma_M} = \underset{\gamma_0}{ArgMax}\frac{\mathbf{Y}^H\left(\mathbb{I}_K - \mathbb{P}_{Col(\mathbb{S}_{-0})}\right)\dfrac{\mathbf{S}_0\mathbf{S}_0^H}{K}\left(\mathbb{I}_K - \mathbb{P}_{Col(\mathbb{S}_{-0})}\right)\mathbf{Y}}{1-\dfrac{\mathbf{S}_0^H}{\sqrt{K}}\mathbb{P}_{Col(\mathbb{S}_{-0})}\dfrac{\mathbf{S}_0}{\sqrt{K}}} \tag{16}$$

Equation (16) is a one dimensional optimization process. Given an initial value $\gamma_0=\gamma_0[0]$, the normalized first order Taylor expansion of the corresponding signal steering vector at $\mathbf{S}_0$ is:

$$\hat{\mathbf{S}}_0\left(\delta\gamma_0\middle|\gamma_0[0]\right) = \frac{\mathbf{S}_0 + j\delta\gamma\mathbf{S}_0^{'}}{\sqrt{1+(\delta\gamma)^2\dfrac{K^2-1}{3}}}\Bigg|_{\gamma_0=\gamma_0[0]} \tag{17}$$

where $\delta\gamma_0=\gamma_0-\gamma_0[0]$. Substitute this approximation into (16); the likelihood function can be re-written as the ratio of two quadratic expressions:

$$\tilde{\mathcal{L}}(\delta\gamma_0) = \frac{\left|\mathbf{Y}_{\perp 0}^H\mathbf{S}_0\right|^2 + 2(\delta\gamma)\operatorname{Im}\left\{\mathbf{S}_0^{'H}\mathbf{Y}_{\perp 0}\mathbf{Y}_{\perp 0}^H\mathbf{S}_0\right\} + (\delta\gamma)^2\left|\mathbf{Y}_{\perp 0}^H\mathbf{S}_0^{'}\right|^2}{\left|\mathbb{P}_{Col(\mathbb{S}_{-0})}\mathbf{S}_0\right|^2 + 2(\delta\gamma)\operatorname{Im}\left\{\mathbf{S}_0^H\mathbb{P}_{Col(\mathbb{S}_{-0})}\mathbf{S}_0^{'}\right\} + (\delta\gamma)^2\left|\mathbb{P}_{Col(\mathbb{S}_{-0})}\mathbf{S}_0^{'}\right|^2}\Bigg|_{\gamma_0=\gamma_0[0]} \tag{18}$$

$\delta\gamma_0$ is obtained by solving:

$$\frac{d\tilde{\mathcal{L}}}{d(\delta\gamma_0)} = 0 \tag{19}$$

Equation (18) yields a quadratic equation having two roots. The solution that produces the maximum value, $\delta\gamma_{0\max}$, is used to set the new initial value for the next iteration: $\gamma_0[1]=\gamma_0[0]+\delta\gamma_{0\max}$. The iteration continues until desired convergence level is attained. At this point, a new iteration process starts to solve for the AOA of the second signal γ_1. This cycle continues until all of the signals AOAs are found within specified error limits. This completes one cycle of iterations. Additional cycles can be performed until all AOA estimations converge to desired levels.

Figure 4 illustrates a scenario with 2 signal AOAs. A linear array with 3 elements is used in the simulation. The input signal contains a DLOS and a multipath signal with SNRs at -18dB and -21dB, AOAs at 30^o and 60^o respectively. The contractive map is incorporated with the alternating projection method to iteratively estimate the signal AOAs. An initial pair for AOAs for the DLOS and multipath is set at (120°, 160°). Holding the multipath AOA at 160°, the first round of iterations leads to a DLOS AOA at 41.3°. Next, holding the DLOS AOA at 41.3°, a sequence of iterations generates an AOA estimation for the multipath at 56.6°. At this point, a new cycle of iterations starts by holding multipath and DLOS AOA constants at their latest values respectively. This cycle of iterations results in DLOS and multipath AOAs at (29.8°, 60.2°), within the allowed error bound of the true AOAs.

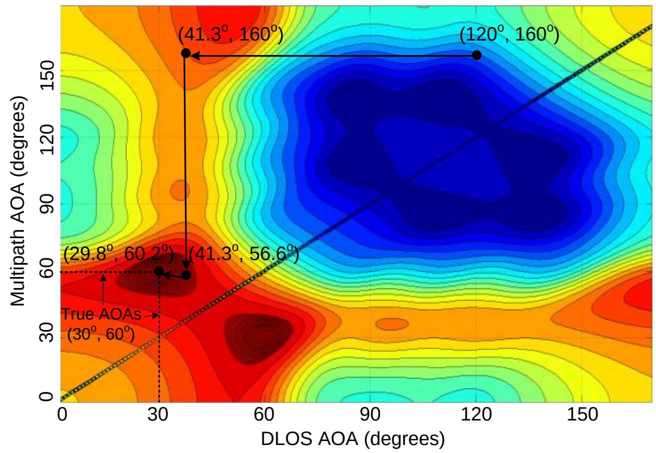


Fig. 4. A 2-dimensional MLE example.

1.4. DLOS Reconstruction Using Pseudo-Inversion

Pseudo-inversion is the operation in which the input vector or the time-domain averaged correlator output vector is projected onto the subspace span by the column of the steering vectors of the signal of interests. We apply pseudo-inversion to recover the DLOS signal after an accurate estimation of its AOA is obtained. Equation (24) is the standard pseudo-inversion operation:

$$\left(S^H(\hat{\boldsymbol{\gamma}})S(\hat{\boldsymbol{\gamma}})\right)^{-1}S^H(\hat{\boldsymbol{\gamma}})\mathbf{Y}(t) \tag{24}$$

If there is only one multipath signal in the input, the first order approximation of the recovered DLOS from (25) can be reduced to:

$$\hat{\rho}_0(t) = A_0 e^{j\phi_0} f_0(\gamma_0,\gamma_1)\delta\gamma_0 + A_1 e^{j\phi_1}\left(1-\frac{\delta\tau}{T_c}\right) f_1(\gamma_0,\gamma_1)\delta\gamma_1 + \varepsilon_0 \tag{25}$$

$$f_0(\gamma_0,\gamma_1) = \frac{\pi\cos\theta_{0,1}\left(\mathbf{S}_1^H\mathbf{S}_0^{'}\right)\sin(\gamma_0)}{K\sin^2\theta_{0,1}} \tag{26}$$

$$f_1(\gamma_0,\gamma_1) = \frac{\pi\left(\mathbf{S}_0^H\mathbf{S}_1^{'}\right)\sin(\gamma_1)}{K\sin^2\theta_{0,1}} \tag{27}$$

where, $\cos\theta_{0,1} = \frac{\mathbf{S}_0^H\mathbf{S}_1}{K}$. For scenarios involving more than one multipath signal, (25) will have additional terms relating to all multipath components.

Note in (25) that the recovered DLOS signal correlation output still contains residuals of the multipath signals and channel noise. The residuals are greatly reduced versions of the original multipath signals and therefore, have much smaller contributions to the multipath induced range error. In Figure 5, three example curves are used to illustrate the performance of the pseudo-inversion in terms of relative multipath power reduction in the recovered signal. Two signals are included in the input: a DLOS and a multipath with SNRs at -18dB and -21dB respectively. The array has 3 linear elements. The relative multipath delay to the DLOS is $0.1T_c$, where T_c=977.5ns is the chip width of the GPS L1 civil code. The three curves are for DLOS AOA at 30^o, 60^o, and 90^o respectively. The horizontal axis is the AOA separation between the multipath and DLOS signal, while the vertical axis is the difference between the multipath and DLOS power ratio (in dB) before and after the pseudo-inversion. Two green horizontal dashed lines are drawn in the figure. The one at 0 dB indicates the boundary, below which there are reductions in the multipath to DLOS power ratio. Clearly, there are two regions in the plots that do not experience multipath to DLOS power ratio reduction. The first region is located near the center of the plots where the multipath AOA is very close to the DLOS AOA. We define a "blind" region in which the multipath AOA and DLOS AOA are so close to each other, that the spatial diversity upon which the contractive map is based loses its effectiveness. Instead of relative power reduction, the pseudo-inversion ends up adding power to the multipath signal. The range of the blind region $\Delta\phi_{blind}$ is one indicator of the effectiveness of the multipath mitigation method presented here. Notice that for the scenario shown in the example, the AOA separation is no more than a few degrees.

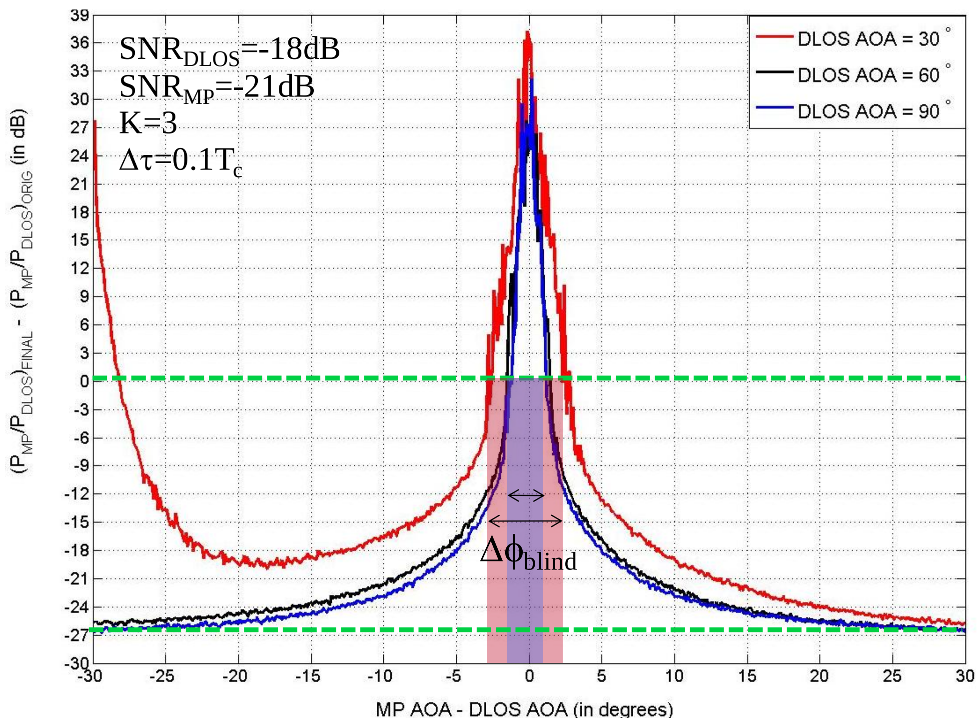


Fig. 5 Difference between the multipath and DLOS power ratio in dB before and after the pseudo-inversion operation.

The second region that does not experience multipath to DLOS power ratio reduction is near the left side of the plot for DLOS AOA at 30^o. This occurred because the multipath AOA is very close to the array axis. Again, the region is rather limited in the sense that it only extends to a few degrees from the array axis. A larger array or a two-dimensional array will further reduce the size of this region.

The green dashed line near the bottom of the plot is the maximum possible multipath to DLOS SNR reduction when DLOS AOA=90^o. We denote the multipath to DLOS power ratio reduction as ΔSNR_{min}. This power reduction is a measure of the best possible multipath mitigation performance for a given array size, DLOS AOA and SNR, and multipath SNR and relative delay time.

1.5. Overall Performance Evaluation

$\Delta\phi_{blind}$, ΔSNR_{min}, and ultimately, the amount of multipath induced range errors provide the overall approach performance measures. Several scenarios are simulated to demonstrate the effectiveness of the method for a 3-element and a 9-element array. For all simulations, the DLOS SNR again is at the nominal value of -18dB.

In the first scenario, one multipath signal is included in the input. Two SNR values are used for the multipath signal: -21dB and -27dB. Figure 6 show the $\Delta\phi_{blind}$ and ΔSNR_{min} values for DLOS AOAs at 20^o to 90^o with 10^o increments. The results show that even for a small array with 3 elements and for a relatively weak multipath nearly 10dB below its DLOS signal with AOA very close to the array axis, we can achieve multipath to DLOS power ratio reduction if the multipath and its AOA separation is more than 15^o. This minimum spatial separation is further reduced if any one of the following parameters increases: the array size, DLOS AOA, and multipath SNR.

Figure 6 also shows that the maximum amount of multipath to DLOS power ratio reduction follows the same trend as that of $\Delta\phi_{blind}$. For a small array with 3 elements, with DLOS AOA very close to the array axis (20^o AOA) and multipath SNR nearly 10dB below the DLOS SNR, the power ratio reduction is nearly 15dB. The amount of the power ratio reduction increases as array size, DLOS AOA, and multipath SNR increases. The best power ratio reduction shown in the figure is over 40dB.

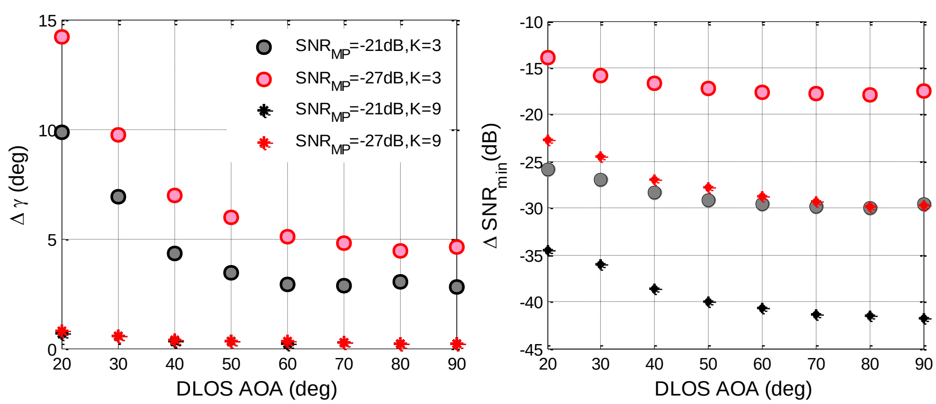


Fig. 6. $\Delta\phi_{blind}$ (top) and ΔSNR_{min} (bottom) for K=3 and 9, multipath SNR=-21dB and -27dB for DLOS AOAs between 20^o and 90^o.

The multipath to DLOS power ratio is directly linked to the multipath induced range error. Figure 7 compares the pseudorange error caused by a multipath before and after the mitigation. The multipath signal in the input is 3dB below its DLOS counterpart. The horizontal axis represents the multipath relative delay time in ns, while the left vertical axis is the pseudorange error in meters before the mitigation algorithm is applied. The plot shows that the errors are in the order of up to 2m for delay time within 10ns. The blue curve and the right vertical axis show the pseudorange error after applying the contractive map and alternating projection method to obtain the DLOS and multipath estimation, and then using the estimation and pseudo-inversion to reconstruct the DLOS signal. The multipath to DLOS power ratio is less than -27dB, more than 24 dB reduction from the original value. This power reduction leads to pseudorange errors in the order of several mm, nearly 400 times reduction in multipath errors.

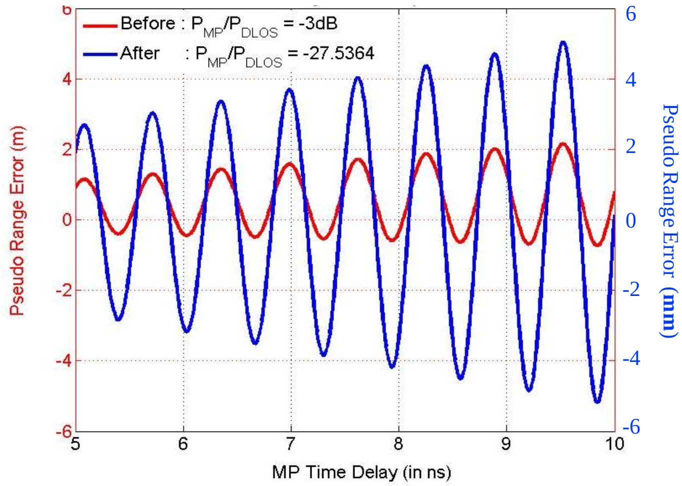


Fig. 7. Pseudorange errors induced by a multipath signal before and after the multipath mitigation algorithm.

The performance analyses presented above are all based on the scenario where only one multipath signal is present in the input. For an input having multiple multipath signals, the scenarios are much more complicated, as one has to take into consideration the relative strength, AOA separations, and delay times among all multipath signals. To demonstrate the effectiveness of the method presented in this paper, without loss of generality, we performed simulations for the scenarios depicted in Figure 8. Two multipath signals are included in the input signal. Multipath 1 AOA is fixed at 100^o, while multipath 2 AOA is placed at 3 different places: 25^o, 75^o, and $125^{o,}$ to reflect a variety of DLOS and multipath AOA separations. The SNR for the DLOS and multipath 1 are -18dB and -24dB respectively. For multipath 2, the SNR assumes the value of -21dB, -27dB, and -35dB at each of the AOA position. The array used in the simulation has 7 elements. Delay time for all multipath signals are 0.1Tc. Figure 9 plots the AOA estimation standard deviations for DLOS (top), multipath 1 (middle), and multipath 2 (bottom) for each scenario simulated. Figure 10 shows the multipath to DLOS power ratio reduction.

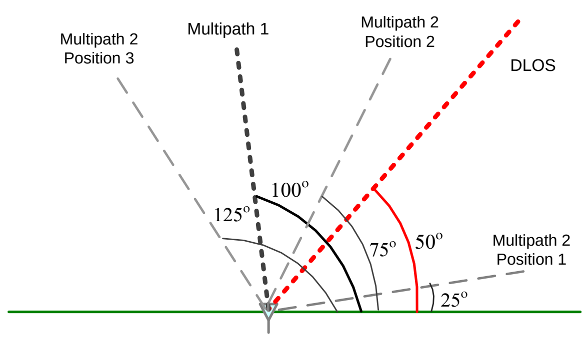


Fig. 8. Simulation scenarios involving two multipath signals. Multipath 1 is at a fixed AOA of 100^o, while multipath 2 AOA is placed at 3 different locations as shown.

Figure 9 clearly shows that for all scenarios simulated, nearly all but one set of standard deviations of the AOA estimations are under 1^o. The SNR of multipath 2 appears to only impact its own AOA estimation. There is no significant effect on the AOA estimations for DLOS and multipath 1. For DLOS and multipath 1, their AOA estimation standard deviations are smallest when all other signals are further away from them. This is a reasonable outcome. For multipath 2, however, our simulation results show that the smallest standard deviation occurs when the signal is located in between multipath 1 and

DLOS. It is important to note here that the biases in the AOA estimations are negligible for all signals.

The average multipath to DLOS power ratio reduction shown in figure 9 clearly indicates that the algorithm involving two multipath signals is less effective compared with the case where only one multipath is involved (see figure 6 for comparison). Again, the algorithm is less effective when the signal AOA is closer to the array axis. Performance improves for multipath with lower SNR.

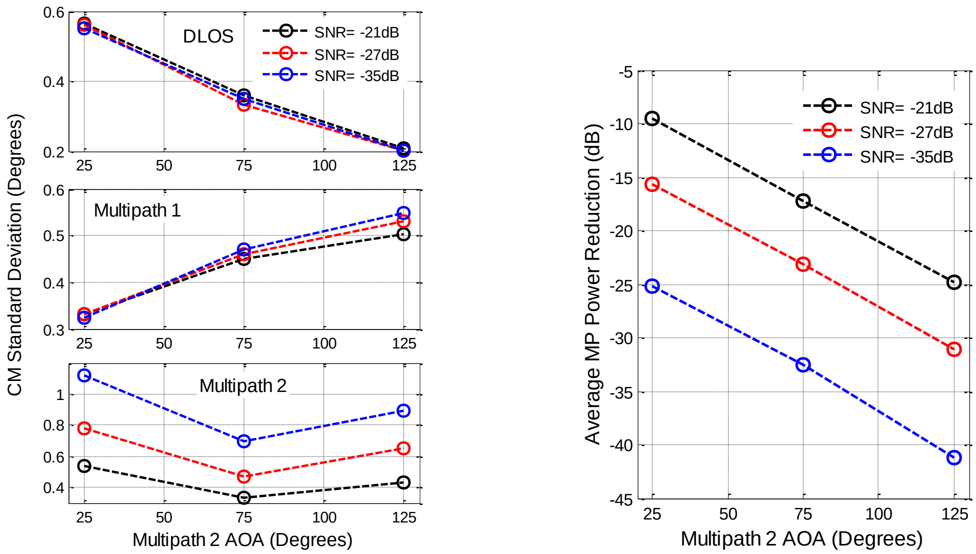


Fig. 9. AOA estimation standard deviation for DLOS (top left), multipath 1 (middle left), and multipath 2 (bottom left) when multipath 2 AOS is at 25^o, 75^o, and 125^o , and average multipath to DLOS power ratio reduction (right) for the 2 multipath scenarios depicted in Fig. 8.

1.6. Conclusions

This project developed a complete set of algorithms that performs detection, estimation, and mitigation of short delay multipath using a linear array of software GPS receivers. Performance evaluations were carried out for inputs containing one and two multipath signals. The results show that for the case of a single multipath, the algorithm can accurately estimate the DLOS and multipath AOA to allow effective reconstruction of the DLOS signals using an pseudo-inversion operator on the input signals. For DLOS signals with AOAs larger than 20^o from the array axis, the algorithm will reduce any single multipath source to DLOS power ratio if the multipath and DLOS AOA separation is larger than 15 degrees. For arrays with reasonable sizes, this separation is further reduced to a few degrees. The amount of multipath to DLOS power ratio reduction is also impressive. Even for a small array with 3 elements, the power ratio reduction is over 15dB for a weak multipath signal. As the array size, multipath power, and DLOS AOA increases, the minimum multipath and DLOS AOA separation decreases and the multipath to DLOS power ratio reduction increases.

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2. Publications

- Published Journal Papers

[1] Brenneman, M., Y. Morton, “Functional bandwidth criterion for adaptive array performance,” *IEEE Trans. Aero. & Elec*., Vol.46, No.3, p1226-1235, 2010.

[2] Brenneman, M., Y. Morton, Q. Zhou, “GPS Multipath Detection with ANOVA for adaptive arrays,” *IEEE Trans. Aero. & Elec.*, Vol.46, No.3, p1171-1185, 2010.

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- Published Conference Papers

[1] Brenneman, M., Y. Morton, “An efficient algorithm for short delay time multipath estimation and mitigation,” *Proc. ION GNSS Conf.*, Portland, OR, Sept. 2010.

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3. Follow-On Uses

- Technology Assists
  - ✓ We worked closely with AFRL-RYRN at Wright Patterson Air Force Base to support the WASPS Program by sharing our understanding and analysis of the multipath errors.

- Technology Transitions: None.

- Technology Transfers: None.

4. Accomplishments and Successes

- Developed an ANOVA-based hypothesis test to effectively detect short delay multipath using a receiver array.
- Developed a contractive map which can efficiently determine a signal AOA without initial knowledge of the AOA.
- Incorporated the contractive map with the alternating projection method to determine multiple and DLOS signal AOAs.
- Developed an inversion algorithm which can reconstruct DLOS signal with very small multipath residual.
- Carried out performance evaluations of all above algorithms to demonstrate the effectiveness and accuracy of the algorithms.

5. Professional Personnel Supported

- Miami University Personnel

  Yu (Jade) Morton, Faculty
  Qihou Zhou, Faculty
  Matthew Brenneman, Research Associate
  Xiaolei Mao, Graduate student

6. Honors and Awards Received

- Yu Morton, Sigma Xi Researcher of the Year Award, Miami University, 2009.

7. Professional Activities

Yu (Jade) Morton
  - Vice Chair, ION Satellite Division (2010-12)
  - President, CPGPS (2011-12)
  - Election Committee Chair, CPGPS (2010-11)

- Vice President, CPGPS (2010-11)
- Outreach Chair, ION (2009-11)
- Eastern Region Member-At-Large, ION (2008-9, 2010-11)
- Chair, Bradley Parkinson Thesis Award Committee, ION (2008)
- Representative, Consortium of Ohio Universities on Navigation & Time keeping (COUNT) (since 2006)
- Chair, Award and Membership Committee, CPGPS (2006-10)
- Technical Committee, IEEE MTT Society, DSP Subgroup (since 2004)
- Session Co-Chair, 2010 CPGPS International Technical Forum
- Session Co-Chair, 2010 ION International Technical Meeting
- Session Co-Chair, 2009 ION Global Navigation Satellite Systems Conference
- Session Co-Chair, 2009 ION International Technical Meeting
- Focused Session Organizer, Advances in Positioning Sys., 2009 Int. Microwave Sym.
- Workshop Organizer, 2008 Miami ECE/ION Dayton Section Symposium
- Session Co-Chair, 2008 ION National Technical Meeting
- Award Committee Chair, 2008 New Navigation Technologies & Innovations Conf., Beijing, China
- Editorial board, Springer journal GPS Solutions (since 2006)
- Associate Editor, IEEE Trans. Aerospace & Electronics (since 2008)